CS-111: Written Assignment 2 Solution

Submission instructions:

Submit your answers to the following questions in a single pdf file on Canvas & Gradescope. Your work is due by **11:59 p.m. on Wednesday, the 22nd of May**.

Questions:

1) Suppose that a 3-bit image has the intensity distribution shown in the Table-1, where intensity levels are integers in the range [0,7]. What is the new intensity distribution after applying histogram equalization/stretching? Your answer should include all necessary calculations and a new intensity table like Table-1. **[10]**

Intensity level r_k	**Pixel number n_k**
$r_0 = 0$	$n_0 = 800$
$r_1 = 1$	$n_1 = 1033$
$r_2 = 2$	$n_2 = 850$
$r_3 = 3$	$n_3 = 648$
$r_4 = 4$	$n_4 = 337$
$r_5 = 5$	$n_5 = 245$
$r_6 = 6$	$n_6 = 122$
$r_7 = 7$	$n_7 = 81$

Table-1. Intensity distribution of 3-bit image.

The discrete form of histogram equalization is

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

$s_0 = 7 * 800/4116 = 1.36 \approx 1$
$s_1 = 7 * 1833/4116 = 3.12 \approx 3$
$s_2 = 7 * 2683/4116 = 4.56 \approx 5$
$s_3 = 7 * 3331/4116 = 5.66 \approx 6$
$s_4 = 7 * 3668/4116 = 6.24 \approx 6$
$s_5 = 7 * 3913/4116 = 6.65 \approx 7$
$s_6 = 7 * 4035/4116 = 6.78 \approx 7$
$s_7 = 7 * 4166/4116 = 7.00 \approx 7$

Intensity level	**Pixel number n_k**
0	0
1	$n_0 = 800$
2	0
3	$n_1 = 1033$
4	0
5	$n_2 = 850$
6	$n_3 + n_4 = 985$
7	$n_5 + n_6 + n_7 = 448$

2) An image has a probability density function (PDF) of *p(r) = 2(1-r)*. We want to transform this image so that its PDF becomes *p(z) = 2z*. Assume continuous images and find the transformation (in terms of *r* and *z*) that would achieve this goal. **[10]**

$$\boldsymbol{P_r = \int_0^r 2(1-w)dw = 2r - r^2}$$

$$\boldsymbol{P_z = \int_0^z 2w dw = z^2}$$

Let $\boldsymbol{P_r = P_z}$, we have

$$\boldsymbol{z = \sqrt{2r - r^2}}$$

3) When we mix blue paint with yellow paint, we get green. But when we project blue light on yellow light, we get brown. How do you explain this contradiction? **[5]**

This is due to **additive** and **subtractive** color mixture.
Combining two colors in subtractive color mixtures means the part of the spectrum not reflected by both is due to being absorbed by either one. Therefore, yellow absorbs blue part of the spectrum while blue absorbs red part of the spectrum and what is reflected by both is green part.
On the other hand, when yellow and blue lights are added, the combination is all wavelengths in these regions leading to brown.

4) Consider a linear display whose red, green and blue primaries have chromaticity coordinates of (*0.5,0.4*), (*0.2,0.5*) and (*0.1,0.1*) respectively. The maximum intensity (defined by *X+Y+Z*) of white is *1000* cd/m^2 respectively. The white point of the display is (*0.33,0.37*). What is the XYZ coordinates of the color generated by the RGB input (*0.5,0.75,0.2*) on this device? **[10]**

We first compute the proportions of RGB components in the intensity of white

$$C_R(0.5,0.4) + C_G(0.2,0.5) + (1 - C_R - C_G)(0.1,0.1) = (0.33,0.37)$$

$$C_R = 0.5,\ \ C_G = 0.3,\ \ C_B = 0.2$$

Since the maximum intensity if white is 1000, the maximum intensities for R,G,B channels are 500, 300 and 200 correspondingly. Then the XYZ coordinate of R,G,B with maximum intensity are

$$XYZ_R = 500 * (0.5,0.4,0.1) = (250,200,50)$$

$$XYZ_G = 300 * (0.2,0.5,0.3) = (60,150,90)$$

$$XYZ_B = 200 * (0.1,0.1,0.8) = (20,20,160)$$

The XYZ coordinates of the color with RGB input (0.5, 0.75, 0.2) is

$$0.5 * XYZ_R + 0.75 * XYZ_G + 0.2 * XYZ_B = \ \ (\mathbf{174, 216.5, 124.5})$$

5) C_1 and C_2 are colors with chromaticity coordinates $(0.33, 0.45)$ and $(0.82, 0.10)$ respectively. In what proportions should these colors be mixed to generate a color C_3 of chromaticity coordinates $(0.55, 0.28)$? If the brightness of C_3 is 90, what are the brightness of C_1 and C_2? **[10]**

$$A(0.33,0.45) + (1 - A)(0.82,0.10) = (0.55, 0.28)$$

There is an issue with the question.
If you compute A using **x** component:

$$0.33A + 0.82(1 - A) = 0.55$$
$$A = 0.551$$

However, if you compute A using **y** component:

$$0.45A + 0.10(1 - A) = 0.28$$
$$A = 0.514$$

Either way will be considered correct.
Then, the brightness of C_1 is

$$90 * A \ = \ 49.59 \ or \ 46.26$$

and brightness of C_2 is

$$90 * (1 - A) = 40.41 \ or \ 43.74$$

6) Consider four neighboring pixels of I denoted by $a = I(x, y), b = I(x, y + 1), c = I(x + 1, y)$ and $d = I(x + 1, y + 1)$. Let us consider a point in the image at location $(x + 0.2, y + 0.8)$. We would like to compute the value of I at P using bilinear interpolation. **[5+3=8]**

a) Write out the equation for this value in terms of *a, b, c,* and *d.*

$$(0.2c \ + \ 0.8a) * 0.2 \ + \ (0.2d + 0.8b) * 0.8$$

Or

$$(0.2a \ + \ 0.8b) * 0.8 \ + \ (0.2c + 0.8d) * 0.2$$

b) What is the degree of this equation?

For location $(x + p,\ y + q)$, the intensity is:

$$(pa \ + \ qb) * q \ + \ (pc + qd) * p$$

In terms of pixel location (p, q), the equation is quadratic.
In terms of pixel intensity (a, b, c, d), the equation is linear.

7) The spectrum of color $C_1 = (X_1, Y_1, Z_1)$ and $C_2 = (X_2, Y_2, Z_2)$ are given by $s_1(\lambda)$ and $s_2(\lambda)$ respectively. Let the color formed by multiplications of the spectrums s_1 and s_2 be s_3, i.e. $s_3(\lambda) = s_1(\lambda) * s_2(\lambda)$. Is it true that the XYZ coordinate corresponding to s_3, denoted by C_3, is (X_1X_2, Y_1Y_2, Z_1Z_2)? Justify your answer with calculations. **[5]**

Let $\bar{x}(\lambda)$ be the standard observer function. Then,

$$X_1 = \int s_1(\lambda)\bar{x}(\lambda)\, d\lambda$$

$$X_2 = \int s_2(\lambda)\bar{x}(\lambda)\, d\lambda$$

Since $s_3 = s_1 \times s_2$, then:

$$X_3 = \int s_1(\lambda)s_2(\lambda)\bar{x}(\lambda)\, d\lambda$$

However, the product of C_1 and C_2 is:

$$X_1X_2 = \int s_1(\lambda)s_2(\lambda)\overline{x^2}(\lambda)\, d\lambda$$

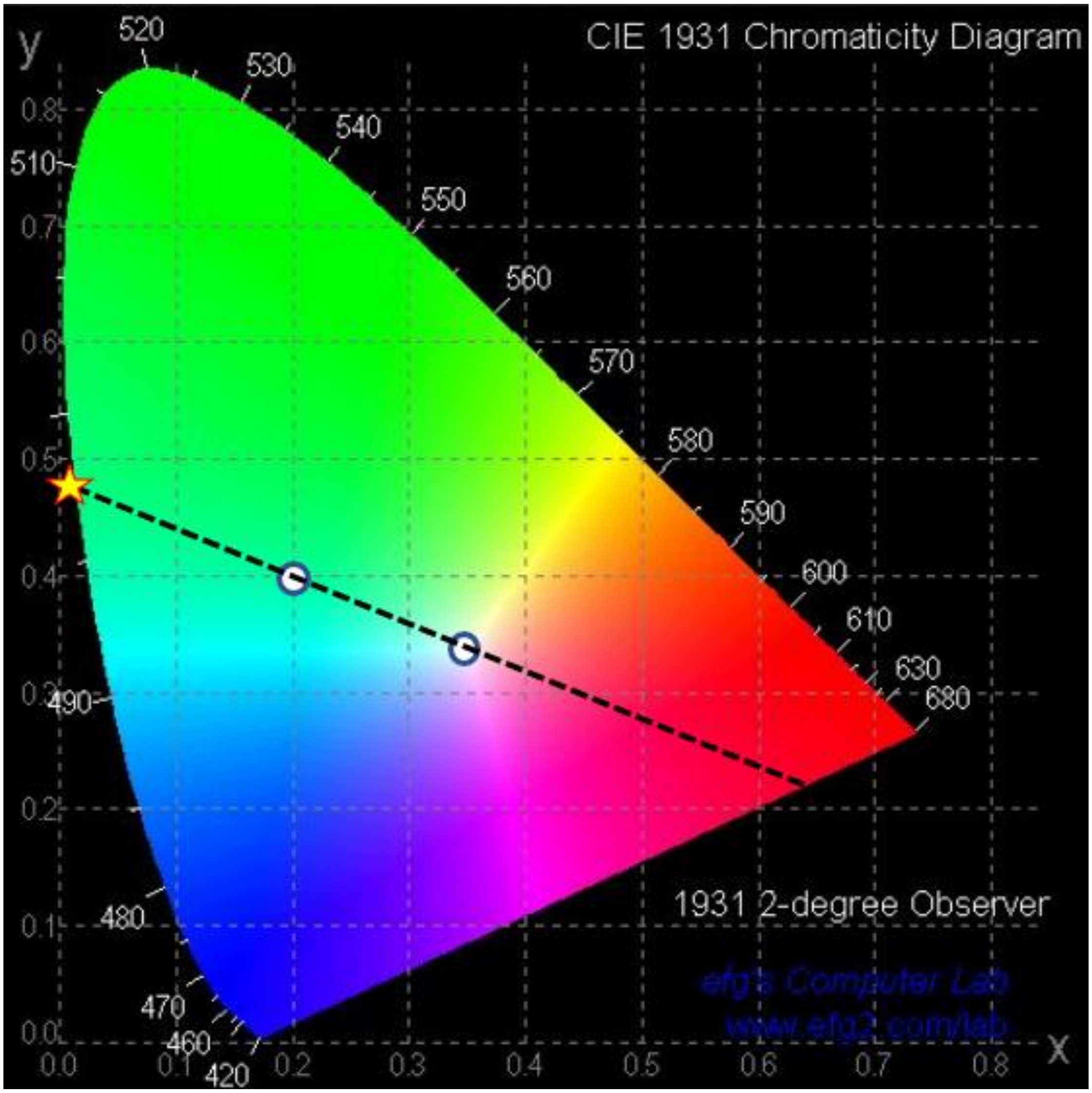
This shows that $X_3 \neq X_1X_2$. We can similarly prove that $Y_3 \neq Y_1Y_2$ and $Z_3 \neq Z_1Z_2$. Therefore, $C_3 \neq C_1C_2$.

8) Consider two spectra, $s_1(\lambda)$ and $s_2(\lambda)$, that are metamers for viewer *A*. However, these two spectra are not a metamer for another viewer *B*. Why does this situation happen? **[3]**

$s_1(\lambda)$ and $s_2(\lambda)$ would be metamers if both observers have **(1)** the same sensitivities of the eye, which **(2)** match perfectly with the standard observer functions $\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda)$.

We know that (2) is not true since later experiments showed that $\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda)$ are imaginary primaries. Similarly, (1) is rarely true; every observer is unique. This phenomenon is known as **observer metamerism**.

9) Consider the color $C = (0.2, 0.4)$ in the chromaticity chart. Find its hue and saturation. Provide the chromaticity coordinate of a color B which when mixed with C will produce white. Find the hue of B. **[2+3+3+2=10]**



In the diagram above, the hue of the color *C* is the point denoted by the yellow star. It is approximately 495-505nm. The saturation is the ratio of the distance of *C* to the white point, to the distance of the white point from the edge of the chart (i.e. the yellow star). In this case, it is approximately 0.40 – 0.45.

The chromaticity coordinates of the color *B* can be any coordinate along the line that extends in the opposite direction i.e. towards the complimentary wavelength, for e.g. (0.4, 0.325) or (0.5, 0.275). Any color along that line will give white when mixed with *C*. The hue of *B* however, is the same as the hue of *C* since the complimentary wavelength lies on the spectral boundary.

10) Answer the following questions about 2D geometric transformations: **[2+2+2+3=9]**

a. What transformation does the following matrix represent?

$$\begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$$

A 2D rotation matrix.

b. Provide a matrix transformation which is the inverse of the transformation in part-(a).

$$\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$$

c. What is the 2D transformation matrix that will reduce an image to half its size?

$$\begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$

d. Provide a single 2D transformation matrix that will reduce an image to half its size, then rotates it by 30 degrees anticlockwise.

$$M = \begin{bmatrix} \cos(30) & -\sin(30) \\ \sin(30) & \cos(30) \end{bmatrix} \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{4} & -\frac{1}{4} \\ \frac{1}{4} & \frac{\sqrt{3}}{4} \end{bmatrix}$$